M/M/1 This is a summary of formulas in the *Elementary Queuing Analysis* lecture note.

$$\text{(utilization) } \rho = \frac{\lambda \text{ (arrival rate)}}{\mu \text{ (departure/service rate)}} \qquad \text{(service time) } T_s = \frac{1}{\mu} = \frac{\rho}{\lambda}$$

Rates are given in "number of requests per unit time," e.g. requests per second.

$$\begin{aligned}
\text{(expected users in system) } q &= w \text{ (num. waiting)} + \rho \text{ (num. serviced)} \\
\text{(total time in system) } T_q &= T_w \text{ (waiting time)} + T_s \text{ (service time)}
\end{aligned}$$

$$q = \frac{\rho}{1-\rho} \qquad w = \frac{\rho^2}{1-\rho}$$

$$T_q = \frac{q}{\lambda} = \frac{1}{\mu(1-\rho)} \qquad T_w = \frac{w}{\lambda} = \frac{\rho}{\mu(1-\rho)}$$

Modifying the mean and variance of PRNG Given $X_1, X_2, \ldots, X_N$ where X_i is a sample from a PRNG, we know that:

$$\bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_i \qquad S^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(X_i - \bar{X}\right)^2$$

What if we want to modify $\bar{X}$ and S from the PRNG by some constant? Let's say we want to offset $\bar{X}$ by a and scale S by b.

$$\begin{aligned}
a + \bar{X} &= a + \left(\frac{1}{N}\sum_{i=1}^{N} X_i\right) \\
&= \left(\sum_{i=1}^{N} \frac{a}{N}\right) + \left(\frac{1}{N}\sum_{i=1}^{N} X_i\right) \\
&= \left(\frac{1}{N}\sum_{i=1}^{N} a\right) + \left(\frac{1}{N}\sum_{i=1}^{N} X_i\right) \\
&= \frac{1}{N}\sum_{i=1}^{N}(a + X_i)
\end{aligned}$$

$$\begin{aligned}
(bS)^2 = b^2 S^2 &= \frac{b^2}{N-1}\sum_{i=1}^{N}\left(X_i - \bar{X}\right)^2 \\
&= \frac{1}{N-1}\sum_{i=1}^{N} b^2\left(X_i - \bar{X}\right)^2 \\
&= \frac{1}{N-1}\sum_{i=1}^{N}\left(bX_i - b\bar{X}\right)^2
\end{aligned}$$

Let $X_i' = a + bX_i$ for $1 \le i \le N$. Show what is $\bar{X}'$ and S'^2. This should give you an idea how to modify the mean and variance of any PRNG.