

Exercise 4 For each natural number n , let $vFib(n)$ be the vector $(Fib(n), Fib(n+1))^T$, that is, the transpose of the vector $(Fib(n), Fib(n+1))$. Clearly, we have the following equation for each natural number n .

$$vFib(n+1) = A \cdot vFib(n), \text{ where } A = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$$

Hence, we have the equation $vFib(n) = A^n \cdot vFib(0)$ for each natural number n . Please use this equation to implement a $\Theta(\log n)$ -time and $\Theta(1)$ -space procedure in ATS that computes the Fibonacci numbers $Fib(n)$.